Mathematical Finance

MAE, Module 4, 2022-2023

Vyacheslav Gorovoy, PhD
NES
vgorovoy[at]nes.ru

Course information

Course Website: https://my.nes.ru

Instructor's Office Hours: by assignment

TAs: TBD

Course description

The course can be considered as a continuation of the Derivatives course with the goal to provide training in financial mathematics, pricing complex derivatives and numerical methods. It consists of the following parts:

- The first part of the course is the theoretical introduction to stochastic calculus and asset pricing based on an equivalent martingale measure approach.
- The second part is an introduction to the Monte Carlo method for pricing derivatives using some of the models, which are widely employed in the industry.
- The third part will cover interest rates models and their application for pricing fixed-income products.

The course is heavily using python, all lecture notes, hws and a project are in Jupyter notebook format.

Course requirements, grading, and attendance policies

Prerequisites:

1. Derivatives
2. Probability theory
3. Python

Grading:

3 homework problem sets	**45% (15% each)**
Class Participation	**10%**
Project	**45%**

Course contents

- Diffusion processes and Black-Scholes model (Lecture notes L4 from Derivatives course)

- Stochastic calculus (Lecture notes L1)
    - Stochastic and Diffusion processes
    - Ito's Stochastic Calculus
    - The backward and forward Kolmogorov equations
    - The Feynman-Kac formula

- Equivalent Martingale Measure (Lecture notes L2)
    - Market price of the risk
    - Choice of numeraire
    - Girsanov theorem
    - Siegel's paradox

- Multi-asset derivatives (Lecture notes L2)
    - Modeling correlation between financial assets
    - Quanto. Application: Hedging risk exposure of oil company

- Models beyond Black-Scholes (Lecture notes L3)
  - Local volatility
  - Stochastic volatility
  - Models with jumps

- Monte Carlo (MC) simulation (Lecture notes L4)
  - Principles of Monte Carlo
  - Pricing Derivatives by MC
  - Variance Reduction
  - Simulation of the CIR Process
  - Heston model.

- Spectral methods in derivative pricing (based on paper: by Fang and others. A Novel Pricing Method for European Options Based on Fourier-Cosine Series Expansions
  - Fourier expansion method
  - Application to Heston Model

- Path-dependent options (Lecture notes L5)
  - Asian options
  - Barrier options

- Interest rates derivatives. (Lecture notes L6)
  - Option on futures. Black's model.
  - Callable and putable bonds.
  - Interest rate swaps
  - Caps and floors

- Interest rates models
  - Short rate models
  - BGM model

- Structured products (SP) (Lecture notes L7)
  - Capital protected
  - Yield enhancement
  - Fixed Income structured notes. Range accrual and Step-up notes
  - Exotic

- ML in Finance
    - Bermudan and American option pricing using NN
    - Volatility surface dynamics based on ML

Description of course methodology

- Lectures
- Homeworks
- Quizzes

Sample tasks for course evaluation

Callable Bond:
Consider a 5-year fixed rate bond with principle of $100 and coupon 4% per year payable semiannually. This bond is callable at 2-year time with a strike price of $100. Assuming that the quoted volatility for the forward yield over a period from 2 to 5 years is 20% and flat yield curve at 4% compounded continuously, compute the current price of the above callable bond.

Course materials

Textbooks and materials

- Lecture notes
- John Hull, “Options, Futures, and Other Derivatives”
- C.W. Oosterlee and Lech Grzelak. Mathematical Modeling and Computations in Finance
- Paper: Fang, Fang and Oosterlee, Kees. A Novel Pricing Method for European Options Based on Fourier-Cosine Series Expansions

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.